Math 116 Section 04

Quiz 7   Name _____

August 09, 2005

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All solutions are to be presented on the paper in the space provided. The quiz is open book. You can discuss the problem with others and ask the TA questions.

Find the following integrals:

(1)

$$
\begin{aligned}
I &= \int e^x \cos x \, dx \\
&= \int e^x (\sin x)' \, dx \\
&= e^x \sin x - \int (e^x)' \sin x \, dx \\
&= e^x \sin x - \int e^x \sin x \, dx \\
&= e^x \sin x - \int e^x (-\cos x)' \, dx \\
&= e^x \sin x + e^x \cos x - \int (e^x)' \cos x \, dx \\
&= e^x \sin x + e^x \cos x - \int e^x \cos x \, dx \\
&= e^x \sin x + e^x \cos x - I
\end{aligned}
$$

Solve for I to get

$$
I = \frac{1}{2} e^x \sin x + \frac{1}{2} e^x \cos x + C
$$

(2)

$$\int \sin 4x \cos 7x \, dx \;=\; \int \frac{1}{2}(\sin(4x - 7x) + \sin(4x + 7x)) \, dx$$

$$=\; \int \frac{1}{2}(\sin(-3x) + \sin(11x)) \, dx$$

$$=\; \frac{1}{2}\left(\frac{\cos(-3x)}{3} - \frac{\cos(11x)}{11}\right) + C$$

(3) Let $x = \dfrac{1}{3}\tan\theta$. Then $dx = \dfrac{1}{3}\sec^2\theta \, d\theta$ and

$$\int \frac{dx}{(1 + 9x^2)^2} \;=\; \int \frac{\frac{1}{3}\sec^2\theta \, d\theta}{(1 + 9\frac{1}{9}\tan^2\theta)^2}$$

$$=\; \frac{1}{3}\int \frac{\sec^2\theta \, d\theta}{\sec^4\theta}$$

$$=\; \frac{1}{3}\int \frac{d\theta}{\sec^2\theta}$$

$$=\; \frac{1}{3}\int \cos^2\theta \, d\theta$$

$$=\; \frac{1}{3}\int \frac{1}{2}(1 + \cos 2\theta) \, d\theta$$

$$=\; \frac{1}{6}(\theta + \frac{1}{2}\sin 2\theta) + C$$

$$=\; \frac{1}{6}(\theta + \sin\theta\cos\theta) + C$$

$$=\; \frac{1}{6}\tan^{-1}(3x) + \frac{3x}{\sqrt{1 + 9x^2}}\frac{1}{\sqrt{1 + 9x^2}}$$

$$=\; \frac{1}{6}\tan^{-1}(3x) + \frac{1}{2}\frac{x}{1 + 9x^2} + C$$